Mechanics Review II

1982 PHYSICS B MECHANICS

(I)

1. The first 10 meters of a 100-meter dash are covered in 2 seconds by a sprinter who starts from rest and accelerates with a constant acceleration. The remaining 90 meters are run with the same velocity the sprinter had after 2 seconds.

   (a) Determine the sprinter's constant acceleration during the first 2 seconds.

   $d_2 = d_1 + v_1(\Delta t) + \frac{1}{2}a(\Delta t)^2$

   $10 = 0 + 0 + \frac{1}{2}a(2)^2$

   $5\,\frac{m}{s^2} = a$

   (b) Determine the sprinter's velocity after 2 seconds have elapsed.

   $V_2 = V_1 + a(\Delta t)$

   $V_2 = 0 + 5\frac{m}{s^2}(2s) = 10\,m/s$

   (c) Determine the total time needed to run the full 100 meters.

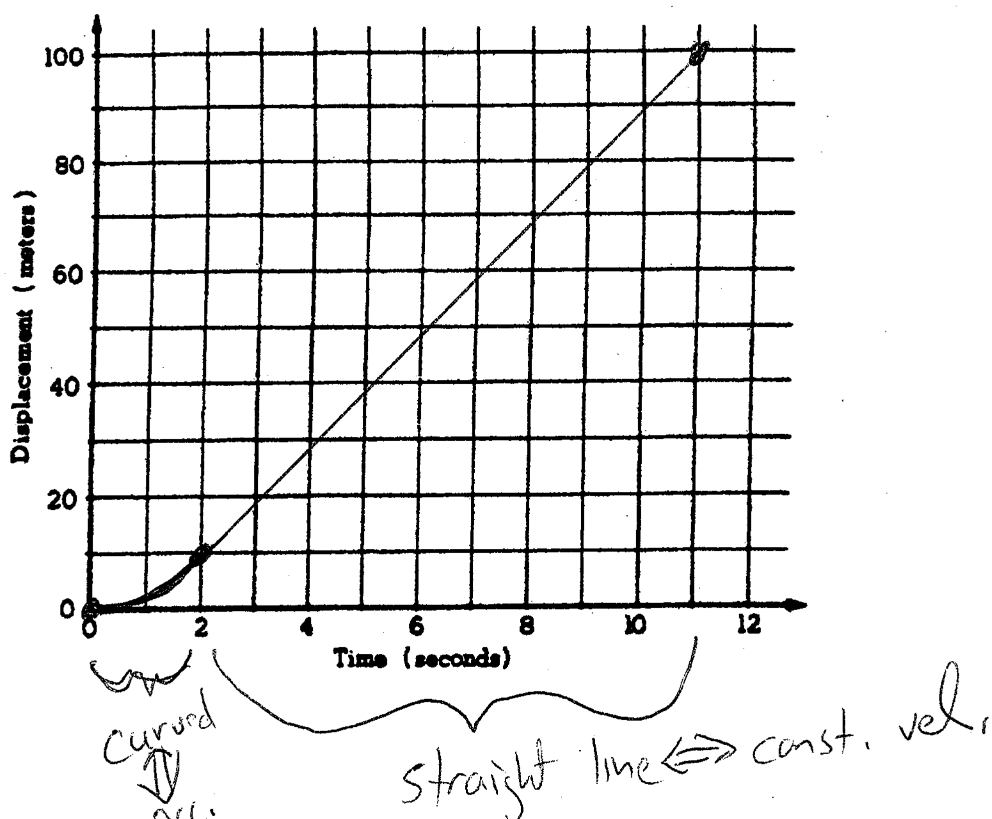
   $\Delta t_{0-10m} = 2s$

   $\Delta t_{10m-100m} = 9s$

   $\Rightarrow 11s$

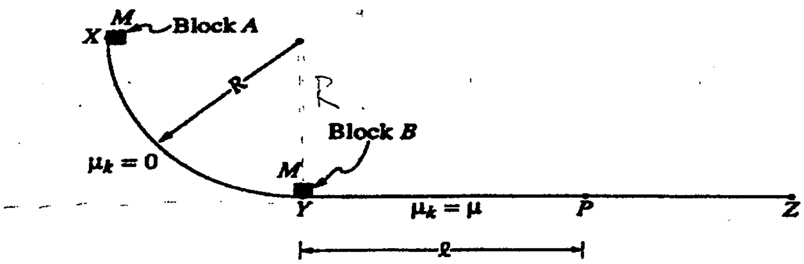
   $v = \frac{\Delta d}{\Delta t}$ $\quad \Delta t = \frac{\Delta d}{v} = \frac{90m}{10m/s} = 9s$

   (d) On the axes provided below, draw the displacement vs. time curve for the sprinter.



1994 PHYSICS B MECHANICS/ENERGY

II



Side View

2. A track consists of a frictionless are *XY*, which is a quarter-circle of radius *R*, and a rough horizontal section *YZ*. Block *A* of mass *M* is released from rest at point *X*, slides down the curved section of the track, and collides instantaneously acid inelastically with identical block *B* at point *Y*. The two blocks move together to the right, sliding past point *P*, which is a distance *ℓ* from point *Y*. The coefficient of kinetic friction between the blocks and the horizontal part of the track is μ. Express your answers in terms of *M*, *ℓ*, μ, *R*, and *g*.

arc and

(a) Determine the speed of block *A* just before it hits block *B*.

$\Sigma E_X = \Sigma E_Y$

$P.E._X = K.E._Y$

$MgR = \frac{1}{2} M v_Y^2 \Rightarrow 2gR = v_Y^2$

$v_Y = \sqrt{2gR}$

(b) Determine the speed of the combined blocks immediately after the collision.

$\Sigma \vec{P}_{before} = \Sigma \vec{P}_{after}$

$M\sqrt{2gR} = 2M v_{after}$

$v_{after} = \frac{\sqrt{2gR}}{2}$

(c) Determine the amount of kinetic energy lost due to the collision.

$\Delta K.E. = K.E._{before} - K.E._{after} = \frac{1}{2}M(\sqrt{2gR})^2 - \frac{1}{2}(2M)\left(\frac{\sqrt{2gR}}{2}\right)^2$

$= \frac{1}{2}M\,2gR - M\frac{2gR}{4} = MgR - \frac{1}{2}MgR = \frac{1}{2}MgR$

(d) The specific heat of the material used to make the blocks is *c*. Determine the temperature rise that results from the collision in terms of *c* and the other given quantities. (Assume that no energy is transferred to the track or to the air surrounding the blocks.)

$Q = c_p m (\Delta T)$

$\frac{1}{2}MgR = c(2M)(\Delta T) \Rightarrow \Delta T = \frac{\frac{1}{2}MgR}{c(2M)} = \frac{gR}{4c}$

(e) Determine the additional thermal energy that is generated as the blocks move from *Y* to *P*.

$Q = W_{friction}$

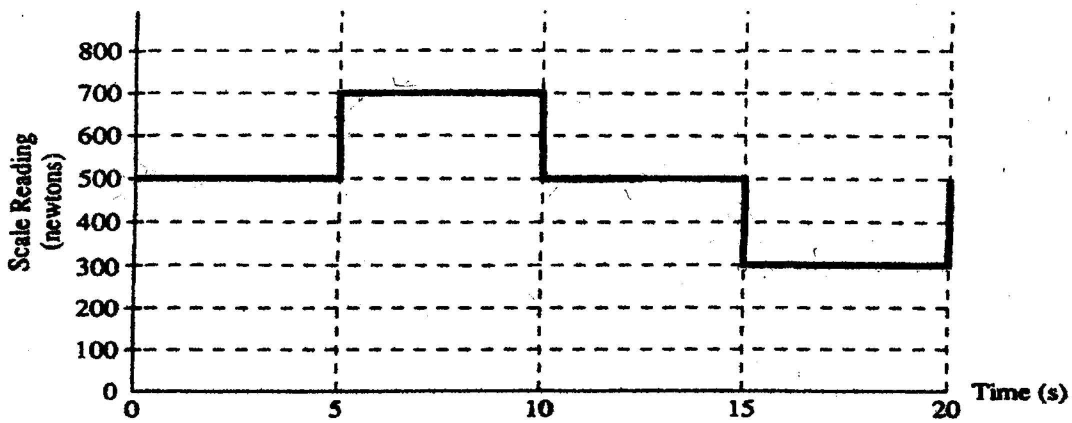
$Q = F_f \cdot d$

$Q = (\mu F_N) \cdot d$

$Q = \mu(2Mg)(\ell)$

$Q = 2\mu Mg\ell$

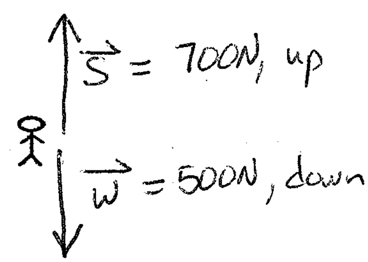
1993 PHYSICS B MECHANICS



III

1. A student whose normal weight is 500 Newton's stands on a scale in an elevator and records the scale reading as a function of time. The data is shown in the graph above. At time t = 0, the elevator is at displacement x = 0 with velocity v = 0. Assume that the positive directions for displacement, velocity, and acceleration are upward.

(a) On the diagram to the right, draw and label all of the forces on the student at t = 8 seconds.



(b) Calculate the acceleration of the elevator for each 5-second interval.

$W = mg$

$\frac{W}{g} = m$

$\frac{500N}{10 m/s^2} = 50kg$

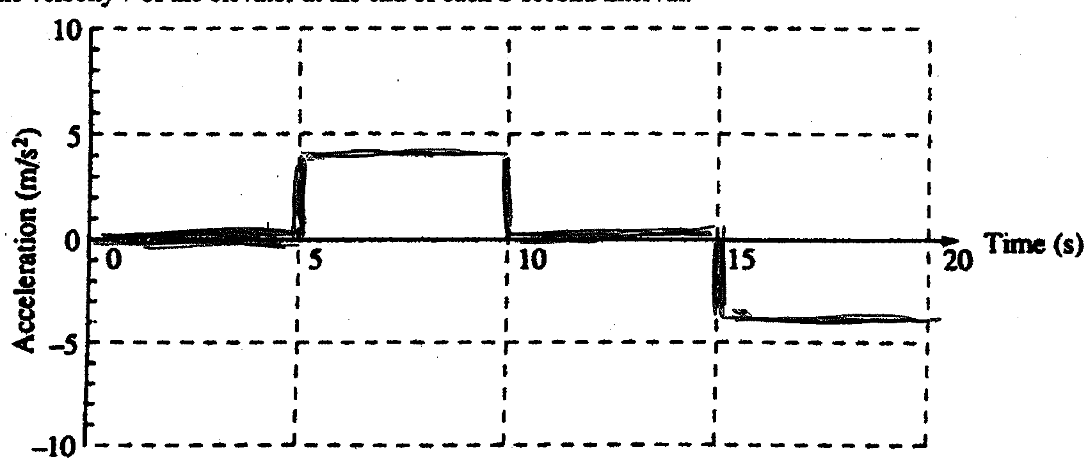
$a = \frac{F}{m} \Rightarrow \frac{200N}{50kg} = 4 m/s^2$ $\quad \frac{-200N}{50kg} = -4 m/s^2$

i. Indicate your results by completing the following table.

Time Interval (s)	0-5	5-10	10-15	15-20
a (m/s^2)	0	$4 m/s^2$	0	$-4 m/s^2$

ii. Plot the acceleration as a function of time on the following graph.

(c) Determine the velocity *v* of the elevator at the end of each 5-second interval.



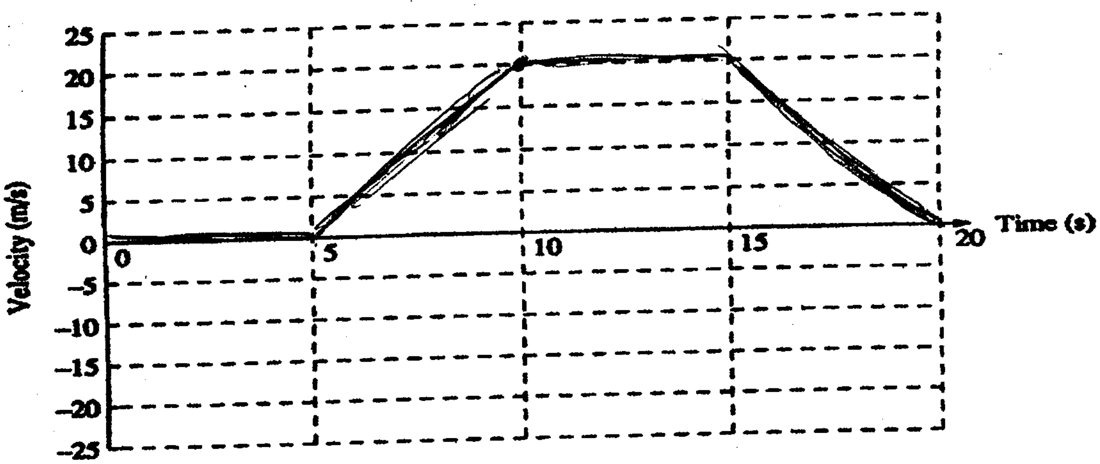
III continued

$4\frac{m}{s^2}(5s) = 20\,m/s = \Delta v$ $-4\frac{m}{s^2}(5s) = -20\,m/s = \Delta v$

i. Indicate your results by completing the following table.

Time (S)	5	10	15	20
v (m/s)	0	+20 m/s	+20 m/s	0 m/s

ii. Plot the velocity as a function of time on the following graph.



(d) Determine the displacement x of the elevator above the starting point at the end of each 5-second interval.

$V_{ave} = 10.0\frac{m}{s}$ $10.0\frac{m}{s}(5s) = 50m$

$20\frac{m}{s}(5s) = 100m$

$100m + 50m = 150m$

i. Indicate your results by completing the following table.

Time (s)	5	10	15	20
x (m)	0	50m	150m	200m

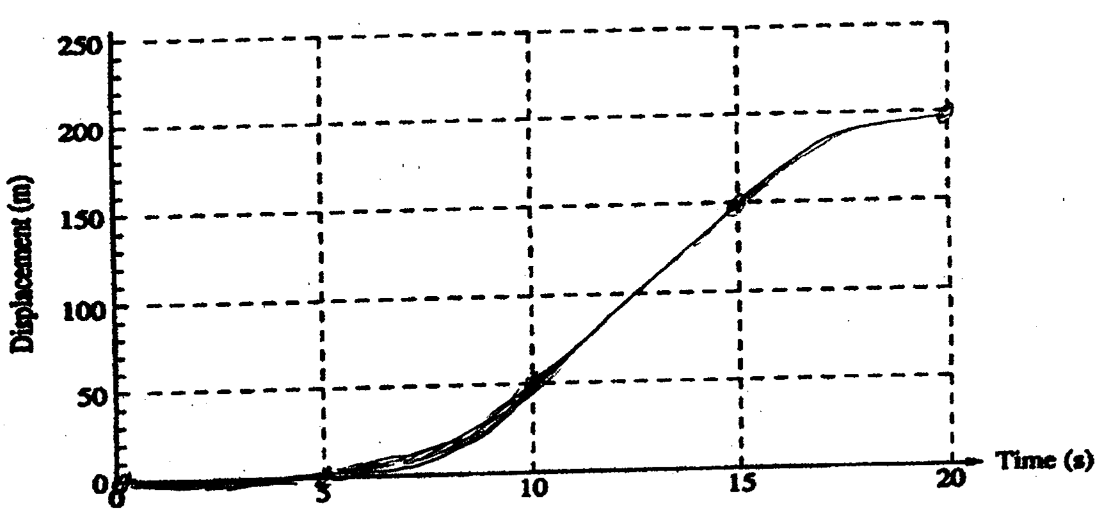
$V_{ave} = 10\frac{m}{s}$

$10\,m/s(5s) = 50m$

$150m + 50m$

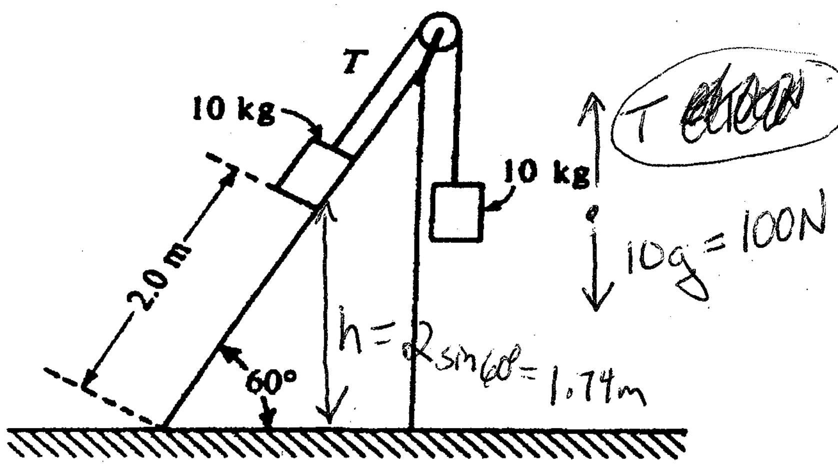
$200m$

ii. Plot the displacement as a function of time on the following graph.



1985 PHYSICS B MECHANICS

IV



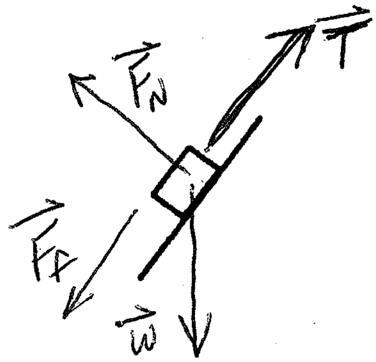
2. Two 10-kilogram boxes are connected by a massless string that passes over a massless, frictionless pulley as shown above. The boxes remain at rest, with the one on the right hanging vertically and the one on the left 2.0 meters from the bottom of an inclined plane that makes an angle of 60° with the horizontal. The coefficients of kinetic friction and static friction between the left-hand box and the plane are 0.15 and 0.30, respectively. You may use $g = 10\ m/s^2$, sin 60° = 0.87, and cos 60° = 0.50.

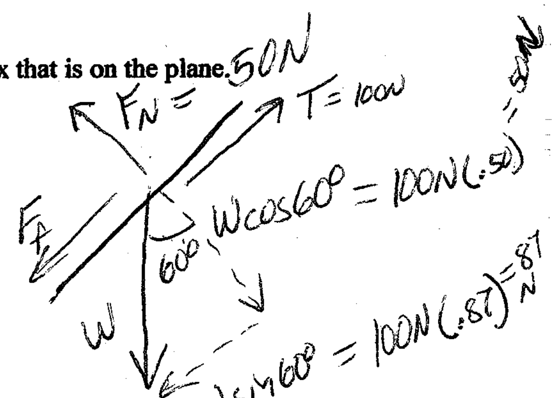
   (a) What is the tension T in the string?

   $\Sigma F = ma$

   $10g - T = 0 \Rightarrow T = 10g = 100N$

   (b) On the diagram below, draw and label all the forces acting on the box that is on the plane.





$\Sigma F_x = 0$

   (c) Determine the magnitude of the frictional force acting on the box on the plane.

$T - F_f - 87 = 0$

$100 - 87 - F_f = 0$

$F_f = 13N$

The string is then cut and the left-hand box slides down the inclined plane.

   (d) Determine the amount of mechanical energy that is converted into thermal energy during the slide to the bottom.

   $Q = W_{friction} = F_f \cdot d = (\mu F_N)(d) = .15(50N)(2m) = 15J$

   (e) Determine the kinetic energy of the left-hand box when it reaches the bottom of the plane.

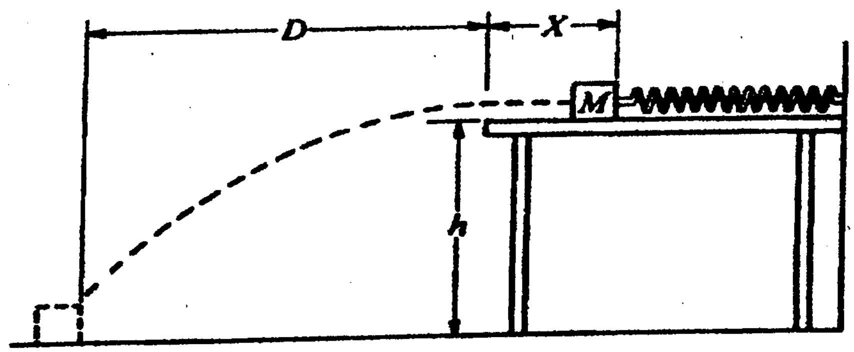
$\Sigma E_T = \Sigma E_B + Q \Rightarrow P.E._T = K.E._B + Q$

$mgh = K.E._B + Q$

$10(10)(1.74) - 15 = K.E. \Rightarrow K.E. = 159J$

V

1986 PHYSICS B MECHANICS



2. One end of a spring is attached to a solid wall while the other end just reaches to the edge of a horizontal, frictionless tabletop, which is a distance h above the floor. A block of mass M is placed against the end of the spring and pushed toward the wall until the spring has been compressed a distance X, as shown above. The block is released, follows the trajectory shown, and strikes the floor a horizontal distance D from the edge of the table. Air resistance is negligible.

Determine expressions for the following quantities in terms of M, X, D, h, and g. Note that these symbols do not include the spring constant.

(a) The time elapsed from the instant the block leaves the table to the instant it strikes the floor

$d_{2y} = d_{1y} + v_{1y}(\Delta t) + \frac{1}{2}a_y(\Delta t)^2$ $\quad h = \frac{1}{2}g(\Delta t)^2$

$0 = h + 0 + -\frac{1}{2}g(\Delta t)^2$ $\quad \frac{2h}{g} = \Delta t^2 \Rightarrow$ $\boxed{\Delta t = \sqrt{\frac{2h}{g}}}$

(b) The horizontal component of the velocity of the block just before it hits the floor

$d_{2x} = v_{1x}(\Delta t)$

$D = v_{1x}\sqrt{\frac{2h}{g}} \Rightarrow$ $\boxed{v_{1x} = \frac{D}{\sqrt{\frac{2h}{g}}}}$

horiz. vel. does not change

(c) The work done on the block by the spring

$K.E._{release} = W_{block}$

$\frac{1}{2}mv^2 = W_{block}$

$\frac{1}{2}M\left(\frac{D}{\sqrt{2h/g}}\right)^2 = W_{block}$

$\frac{\frac{1}{2}MD^2}{2h/g} = W_{block}$

$\boxed{\frac{gMD^2}{4h} = W_{block}}$

(d) The spring constant

$P.E._{spring} = W_{block}$

$\frac{1}{2}kX^2 = \frac{gMD^2}{4h}$

$k = \frac{gMD^2(2)}{4hX^2} = \boxed{\frac{gMD^2}{2hX^2}}$